

Coevolution of strategies in n -person Prisoner's Dilemma

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Abstract

The evolution of strategies in the iterated n -person Prisoner's Dilemma game is studied in various types of models. By varying the payoff parameters and other characteristics of the models, we investigate some circumstances under which cooperative behaviour evolves, both in a mean-field situation (where all interact with all) and in a spatially extended system on a lattice using a cellular automaton dynamics (with only local interactions). In one class of mean-field models, cooperative behaviour may dominate in a dynamics that avoids less cooperative stable fix points. We also present and briefly discuss models that use finite automata as representation for the strategies.

Introduction

The evolution of cooperative behaviour has been studied extensively, through the use of the two-person Prisoner's Dilemma (PD) game as a model for interaction between individuals. In such models reciprocity and 'kin selection' are examples of mechanisms that allow for cooperation to be established. In multi-person games, the problem of avoiding exploitation, or free riders, is more difficult, and cooperation may be harder to achieve.

It is well known that, in the iterated PD game without noise, there are simple strategies that are capable of establishing cooperative behaviour^{1,2}. Also when noise, in the form of misunderstanding or mistake, disturbs the game, there are strategies that may correct for the mistakes, or may deal with the misunderstandings, in order to re-establish cooperation^{3,4,5,6,7}. This seems to be the case for a variety of model settings and parameter choices.

It turns out that there may be different mechanisms that are active in deciding which cooperative, or in some cases defecting, strategy that will dominate in a certain environment. This depends on,

for example, if the model is of the mean-field type, i.e., that all individuals interact with all, or if there is a spatial distribution of individuals where the game is played locally.

In the spatially extended models, coevolutionary dynamics may result in cooperative behaviour of very simple strategies that are maintained due to a form of (localized) kin selection^{8,9,10}. In some cases complicated spatio-temporal patterns like spiral waves may be sustained due to group selection, in which different types of groups compete for space. These types of mechanisms seem to be less common in mean-field models, where most of the cooperative strategies depend on reciprocal altruism like the Tit-for-tat strategy. Other mechanisms may include choosing the partner to play with¹¹.

Much more complex games are given by the multi-person versions of the Prisoner's Dilemma¹². In these games reciprocity may be less advantageous to use, since you are not only punishing the defector but also all others that did cooperate. Therefore, the multi-person game provides us with an interesting problem to be used in coevolutionary dynamics modelling.

In social and natural systems the action of an individual often affects a number of other individuals, and there are numerous examples of situations where “free riders” or defectors take an advantage of others cooperating for a common good³. This type of problem has been recognized as “the tragedy of the commons”, an expression coined by Hardin¹³. Many environmental problems, where, e.g., the costs for exploiting the assimilation capacity of ecosystems are not internalized in the economy, are of this type. The largest problem of this type may be the increase of the greenhouse effect due to emission of fossil carbon into the atmosphere. This situation is even more problematic since some of those “participating” in the game (or affected by the choice of action) are not yet born.

When several agents are exploiting a common resource, such as a fish population in a lake, there is an almost inevitable risk that overexploitation will result in a lower overall yield. In problems of this type there is no “best” solution on the individual level (unless individual optimization also takes into account how the choice of action influences the future composition of strategies in the population), but what is good depends on the behaviour of the others. In such situations coevolutionary dynamics is one approach to investigate under what circumstances individual strategies lead to the overall best solution (highest total yield).

N-person games may serve as one basis for understanding aggregation to higher system levels in evolutionary systems (in biology or society), e.g., the transition from unicellular to multicellular organisms, the formation groups in societal systems, the division of labour in predator inspection¹⁴. Evolutionary transitions involving the aggregation of previously separate

units often depends on cooperative behaviour that is made permanent as a result of the aggregation. For a discussion on major evolutionary transitions, we refer to Maynard Smith and Szathmary¹⁵.

A number of evolutionary models using the (iterated) n -person Prisoner's Dilemma game (n -PD) have been proposed (as well as models on social dilemmas based on agents trying to predict their future benefits¹⁶). A variation of an iterated n -person PD game on a lattice is given by a local game where an individual's action simultaneously is used in all local groups that she is participating in, implying that the iterated game is global — a certain action in one lattice site may spread over the whole system. This type of n -PD game on a lattice was studied by Matsuo and Adachi^{17,18,19}, and large scale cellular automata simulations using this game was performed by Albin²⁰. In the spatially extended models used in the present study we use the approach by Matsushima and Ikegami²¹, in which all groups are closed, i.e., each player chooses a unique action for each of the local groups.

Several ways of representing strategies have been suggested and put into evolutionary models. In this paper we focus on a very simple strategy set that has been previously analysed by, e.g., Boyd and Richerson²² and Molander²³. The results of our simulations add new insight to their analysis. For example, the behaviour in the coevolutionary dynamics is not always well described by the fixed point characteristics. More advanced types of strategies for the n -PD are finite memory strategies^{24,25,26}. A more general class of strategies is given by finite automata^{19,21}, and we propose such a representation and apply it to the two types of coevolutionary models studied: the mean-field models with global interactions and the cellular automata models with local interactions.

The n -person Prisoner's Dilemma

In a single round of the n -person Prisoner's Dilemma game, n players simultaneously choose an action, cooperate or defect. Depending on the number i of others cooperating, you receive the score $V(C | i)$ when you cooperate and the higher score $V(D | i)$ when you defect. The scores V increase with an increasing number of cooperators, and also the total score given to all players increases if one player switches from defection to cooperation. To summarize:

$V(C | i)$: score for playing C when i others cooperate.

$V(D | i)$: score for playing D when i others cooperate.

$V(D | i) > V(D | i - 1)$, and

$$V(C | i) > V(C | i - 1), \quad i = 1, \dots, n-1. \quad (1)$$

$$V(D | i) > V(C | i). \quad (2)$$

$$(i + 1)V(C | i) + (N - i - 1) V(D | i + 1) > i V(C | i - 1) + (N - i) V(D | i). \quad (3)$$

In this paper we shall assume that the scores V can be calculated as a linear combination of the scores against the other players in $(n-1)$ ordinary 2-player PD games. Note that this is still an n -person game since the same action is performed simultaneously in all games.

In the two-person game, the scores are R (reward) for mutual cooperation, T (temptation score) for defection against a cooperator, S (sucker's payoff) for cooperation against a defector, and P (punishment) for mutual defection, with the inequalities $S < P < R < T$ and (usually) $R > (T+S)/2$. It is known that in evolutionary models for the two-person PD on a lattice, there are only two independent parameters⁸. This also extends to the n -person game on the lattice, and we shall use fixed values on R and S in this study. Therefore we assume that $R = 1$ and $S = 0$, while $1 < T < 2$ and, $0 < P < 1$. Also for the mean-field models we use this parameter space (together with a third independent parameter as a growth constant in the population dynamics equations). Then we define the score functions V as follows (where we have divided by $n-1$ in order to make it easier to compare results from different group sizes).

$$V(C | n_C) = \frac{n_C}{n-1} \quad (4)$$

$$V(D | n_C) = \frac{T n_C}{n-1} + \frac{P(n - n_C - 1)}{n-1} \quad (5)$$

In the single round game, the rational choice of action is to defect, leading to all players in the group defecting and scoring only $P < 1$ instead of 1 which they get if they all cooperate. If there is a high probability that the group will play again, we have the iterated n -person Prisoner's Dilemma, and then cooperation may develop under some circumstances. The purpose of this paper is to study some coevolutionary models and to discuss some of the conditions for the evolution of cooperation. For simplicity, since we want to avoid simulating the game, we assume the games to be infinitely iterated, and we use the average score per player and round as a fitness variable for the selection in the population dynamics.

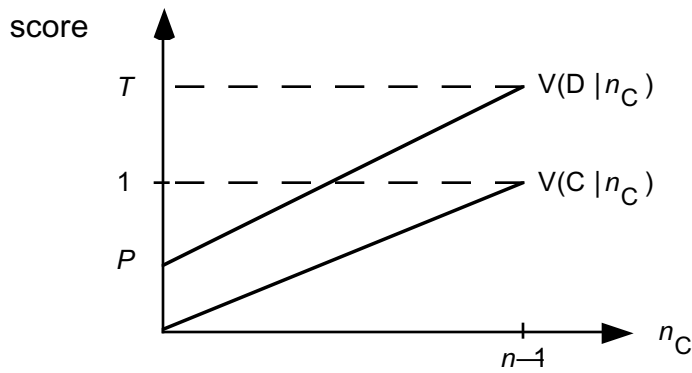


Figure 1. The scores V depend on the number of cooperating opponents n_C in the group playing the n -person PD game. This is a linear generalization of the two-person game.

Strategy types used in the n -person Prisoner's Dilemma

Several different types of strategies have been tested in the n -person Prisoner's Dilemma. Here we will briefly describe the most commonly used classes of strategies. The strategies can be divided into different classes, depending on how they are constructed. The characteristics that divide the strategies into different classes are:

- Whether the strategies are deterministic or probabilistic, that is whether a strategy always plays C (or D) for a given history or whether it does it with a certain probability. (In this study we have only included deterministic strategies.)
- Whether the strategies have memory, i.e., whether an individual using a strategy remembers the decisions that itself and other individuals in the same group made previous rounds of the game.
- Whether a strategy has internal states or not. Strategies with internal states can be described by finite automata.

Strategies without memory

If the strategies are only AllC (always cooperate) or U (unconditional defection), i.e., they have no memory, the n -person game is equivalent to the 2-person game for the linear payoff functions assumed in the previous section. Then it is well known that, regardless of parameter choice, any mean-field evolutionary model evolves to a population completely dominated by unconditional defectors U. On the other hand, if the game is played locally on a lattice using a cellular automaton updating rule, coexistence in various forms of spatio-temporal patterns is possible between the C and D strategies for some parts of parameter space^{8,9,10}. This restricted set will not be dealt with further in this paper.

The simple set of the S_k strategies

A natural extension of the strategy set is to allow "triggering" strategies, that switches to defection if sufficiently many of the others in the group defect. These strategies can be viewed as having a one step memory, remembering the number of cooperators in the previous round. Suppose that the game is played in groups of n players. Then the strategy set $\{ S_k \}_{k=0, \dots, n}$ can be defined by

S_k : Cooperate (C) if at least k others do, otherwise defect (D).

For example:

S_0 = Always cooperates.

...

S_{n-1} = Cooperates if all others do.

S_n = U = Unconditional defection.

These strategies can be interpreted in two ways. The first fits into a single round game, where it can be assumed that the players negotiate to find who shall defect and who shall cooperate. Then they are forced to do what they have promised. The second interpretation involves the iterated game where the players start with assuming that the others will cooperate, but in the following rounds they adapt their actions according to their strategies, i.e., if too many of the others defect they also switch to defection (which S_n , or U, does anyway). This leads to the game settling down to a certain number of cooperators, and if the stationary score decides the fitness of the players, the two views are equivalent.

Finite memory

Hauert and Schuster²⁵ have looked at more general and probabilistic strategies with memory 1, which means that individuals remember the decisions of all members in the group the previous round and act with this in mind. Each strategy can be represented by a transition matrix in which each element represents the probability p_x for the player to cooperate (play C) given a certain history X (and $1-p_x$ is the probability for the strategy to defect). Numerical simulations done by Hauert and Schuster for 3 and 4 players in each group show that cooperative solutions to the game exist, though they seem not to be evolutionarily stable. Defecting strategies can invade a cooperating population but cooperation may be reestablished after several generations.

Another example of a memory one strategy class was proposed by Akimov and Soutchanski²² who used strategies that takes its own previous choice and the majority choice of the other players as input.

In an evolutionary model it may be desirable to allow the memory capacity to increase by mutations in order to allow for more advanced strategies to evolve. This approach has been used before, both for finite memory strategies^{4,8} and for the more powerful representation of finite automata^{27,6} described next.

Finite automata

We have studied strategies that have internal states and that can be described by finite automata (FA). The nodes represent the possible internal states (and corresponding actions, C or D) for an individual using the strategy, and the transition arcs show how the player switches between internal states. In Fig. 2, the structure of such an FA strategy for the three-person PD game is shown.

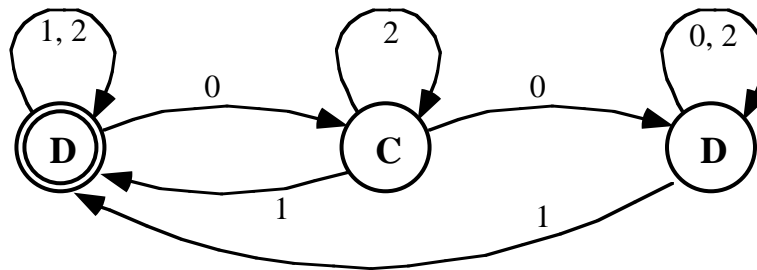


Figure 2. This is an example of a non-trivial finite automaton strategy, for the three-person PD game, that evolved in one of the simulations.

The start node is marked by a double circle. The next action depends on how many other players in the group that played C last time, and the arcs are labeled by this number. For the 3-player strategy shown in Fig. 2, the player starts with cooperation, but if no others cooperate she switches to the second node (determined by the arc labeled 0 for no cooperators) to cooperate in the next round. Then the player will stay in the second node cooperating as long as all others do, etc. (The strategy described by this automaton has no finite memory representation.)

In the simulations we have used the following simple mutations: alter node label, move the address of an arc, and change start node. In order to allow for evolution from simpler to more complex strategies, we have also used mutations that add new nodes. We construct such a growth mutation so that it is neutral, i.e., the arcs connecting the new node are introduced in such a way that the resulting strategy has exactly the same behaviour as before. (This is done by splitting a node that has two arcs leading to it. One of the arcs leads to the old node while the other one leads to the new node, which is a copy of the old one including both the label as well as all arcs leaving the node.)

Evolutionary modelling

The iterated n -person Prisoner's Dilemma is used as a description of the interaction within groups of n individuals picked from a large population of N individuals. We use two different types of dynamics resulting in two different classes of coevolutionary models: (i) *mean-field* models in which all combinations of individuals are used in each generation, and (ii) *cellular automata* (CA) models in which local groups are playing the game on a square lattice. In both cases we illustrate the evolutionary behaviour both for the simple S_k strategies and for the finite automata strategies.

Mean-field models

In the mean-field model we assume a population of N individuals (here $N = 1000$) and in each generation all possible combinations of the present strategies are used in the formation of the groups of n players. The infinitely iterated n -PD is played in every group. Then the difference between the score for each strategy type and the average score in the population decides the change in population density for the different strategies. The change, from one generation to the next, in population density x_i for strategy i ($i = 1, \dots, M$, where M is the number of different strategies present), can then be written

$$x'_i = x_i + d x_i \left(\sum_{j_1, j_2, \dots, j_{n-1}} \left(\prod_{k=1}^{n-1} x_{j_k} \right) s(i, j_1, \dots, j_{n-1}) - \sum_{j_1, j_2, \dots, j_n} \left(\prod_{k=1}^n x_{j_k} \right) s(j_1, \dots, j_n) \right), \quad (6)$$

where d is a growth constant. Here $s(i, j_1, \dots, j_{n-1})$ is the score for a player of strategy i playing in a group with the strategies j_1, \dots, j_{n-1} , and hence the second sum is the average score. These equations conserve total populations size; here $\sum x_i = 1$. The number M of different strategies may change due to mutations, which are randomly applied after each generation (step) of the equations (6). Even if the x_i 's are continuous variables, the model has a discrete component in that a certain population size N is assumed, implying that one individual is given by $x = 1/N$. If any x_i falls below this value, the corresponding strategy is extinct and removed from the dynamics. (The extinctions may change the total population size, and therefore the population is normalized to 1 after each generation.) The described dynamics means that generations are non-overlapping and that reproduction is asexual.

Cellular automata models

For the spatially extended system we have chosen the synchronous and local updating rule of a cellular automaton, with one player in each cell (or lattice site) on a square lattice with periodic boundary conditions. The world size used is 128x128. Groups are formed locally to play the

five-person PD game, so that each group has a player in the center and the four nearest neighbours. This means that each individual participate in five different groups (and games).

This leads to a CA with a next-nearest neighbourhood interaction. The average score for each individuals is calculated and compared to the nearest neighbours' scores. In each five-cell neighbourhood the highest scored individual will put its offspring in the middle cell, see Fig. 3. A small random number is added to each individual's score in order to break ties. The offspring inherits the parent's strategy, possibly altered by mutations. (Of course, other local group formations could also be chosen. For example, local three-player groups have been used by Matsushima and Ikegami²¹.)

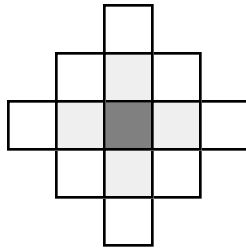


Figure 3. In the cellular automaton model each individual participates in five different five-person PD games, involving up to next-nearest neighbours. Then the player with the highest score in the five-cell neighbourhood puts its offspring in the middle cell. This rule is applied in parallel over the whole lattice.

Mathematical analysis of the S_k strategies

In this section we review and discuss some results of the extensive study by Boyd and Richerson²² on the S_k strategies. Even though they have some restrictions on their payoff parameters, parts of their results can be used in a qualitative way for discussing some of the behaviour we observe in the coevolutionary models.

Boyd and Richerson argue that the conditions that allow the evolution of reciprocal cooperation become extremely restrictive as group size increases. This statement is based on their investigation of strategies of the type S_k playing the iterated n -person Prisoner's Dilemma. They sample groups of n individuals from a large population and let them play the iterated game with a varying length determined by a parameter w , which is the probability for a sampled group to interact another time. That means that the expected number of iterations in a group is $1/(1-w)$. In the limit $w \rightarrow 1$, we have the infinitely iterated Prisoner's Dilemma as we have discussed before.

A population in which unconditional defectors are dominant can resist invasion by any reciprocating strategy. This is true for all values of w (except $w = 1$). This can be understood because if S_n is dominant, a strategy S_a ($a < n$) will most likely meet $n-1$ unconditional defectors

and will get a lower score than them. Hence, individuals using strategy S_a cannot invade a population of unconditional defectors. For the infinitely iterated game, which corresponds to $w=1$, an individual using a reciprocal strategy (except S_0) in a group of $n-1$ defectors will get the same payoff as the unconditional defectors and can therefore (initially) increase by a genetic drift (and then, for example, may S_{n-1} increase by its own cooperative and reciprocating behaviour).

A population in which strategy S_{n-1} dominates can resist invasion by unconditional defectors if, and only if, w is sufficiently large. S_{n-1} is the only strategy that has this property of resisting invasion from unconditionally defectors. With linear payoff, the domain of attraction of S_{n-1} becomes rapidly smaller as the group size increases. The domain of attraction of S_{n-1} is the set of initial strategy combinations that end up in an equilibrium with S_{n-1} dominant. There is a smallest initial frequency of S_{n-1} for which the population eventually will end up with only individuals using S_{n-1} . This frequency tends to one as the group size increases. If the initial frequency of S_{n-1} is lower than this value, the population will end up with only unconditional defectors.

In a population composed only of a strategy S_a ($0 < a < n-1$) and the unconditional defector U, there exist one evolutionary stable equilibrium if w is large enough. This stable equilibrium will eventually evolve if the initial frequency of S_a is larger than a value that depends on the parameters in the prisoner's dilemma game and on the group size. This initial frequency increases with the group size. If the initial frequency of S_a is less than this critical value the population will eventually consist only of unconditionally defectors. A population at the stable equilibrium involving two strategies S_a ($0 < a < n-1$) and U can resist invasion from other rare strategies S_b ($b \neq a$).

The overall conclusion that can be drawn from Boyd and Richerson's paper is that it is more difficult for cooperation to evolve in larger groups. Although, they only show that this is true when evolutionary stable states involve only one or two types of strategies. Our numerical simulations show that there exist other evolutionarily stable states involving more than two strategies. Therefore, there may be parameter values for which this conclusion is not true. As we shall see in the simulation examples, there are also other dynamical effects that further complicate this picture.

Boyd and Richerson have also studied the case where the groups are not formed randomly, but instead reciprocal strategies are more likely to meet each other than just by chance. In this case, reciprocating strategies are more likely to increase. This kind of social interactions could arise if individuals tend to interact with genetic relatives. Such a modification of the game will not be

considered further in this article. Still, it is worth to notice that the CA models may involve such effects, since the local interactions may imply that there is a high probability that your relatives are in the group.

Simulations and results

Here we shall briefly show our preliminary results for the mean-field and the CA coevolutionary models, using both the S_k strategies and the finite automata representation. A more thorough investigation will be reported elsewhere.

The simple set of the S_k strategies

Mean-field model

It is known from the results of Boyd and Richerson that in the S_k strategy set there are stable fixed points involving the U strategy (unconditional defection) and one of the S_k ($k < n - 1$). In a coevolutionary model, however, the mutations may form a distribution of strategies that avoids the stable fix points in establishing a population of cooperating S_k strategies ($k < n$). Depending on the distribution of these strategies the mutant U strategy may increase and in some cases the dynamics is caught by a fixed point with mixed actions, or the population may swing back to a fully cooperative (meta-stable) state again. An example of such a pattern is shown in Fig. 4 below, where oscillations between cooperative and defective dominance is seen for the first 120,000 generations, but after that the system settles to a stationary state involving all strategies except S_4 . Two of them (S_0 and S_2) are maintained due to mutations only, while there is a fixed point of the mutation-free dynamics involving the three strategies U, S_3 , and S_1 . This positively answers the question by Boyd and Richerson²² of the existence of fixed points with more than two strategies in this dynamics, and it contradicts the claim by Molander²³ that such a fixed point does not exist.

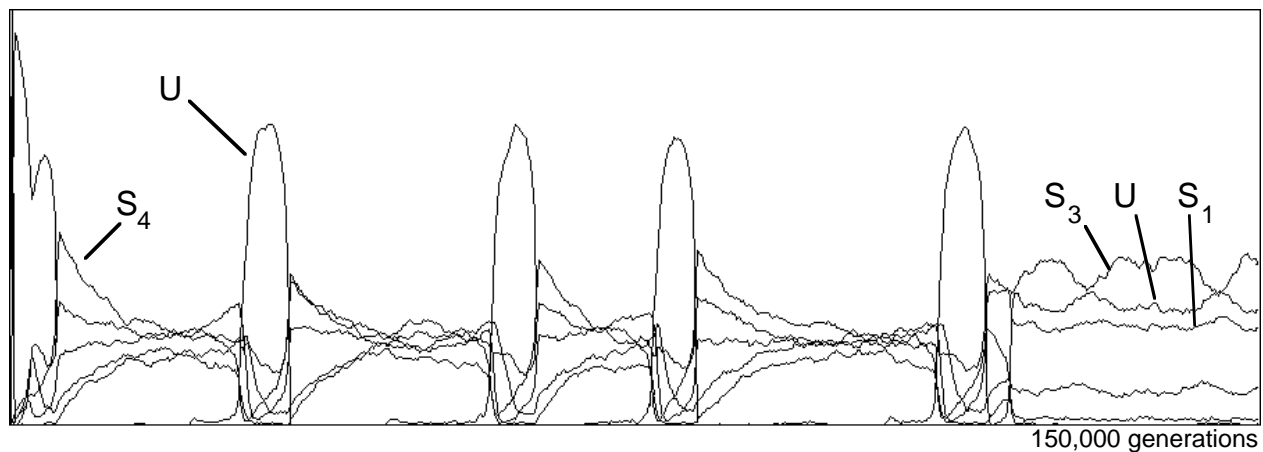


Figure 4. For the simple strategy set, there are payoff parameters leading to oscillatory behaviour between cooperating strategies and the unconditional defectors. Here we have used the 5-player game in the mean-field dynamics, with the parameter values $T=1.5$, $P=0.2$, $p_{\text{mut}} = 0.0001$, $d=0.04$, and $N=1000$.

In order to illustrate how the behaviour in this simple system depends on the payoff parameters, we have run a series of simulations recording the average score for the 4-person PD as functions of T and P . The statistics shown in Fig. 5, illustrate that there is a non-trivial pattern of regions where cooperation is established. From this picture we can sketch the phase diagram in Fig. 6.

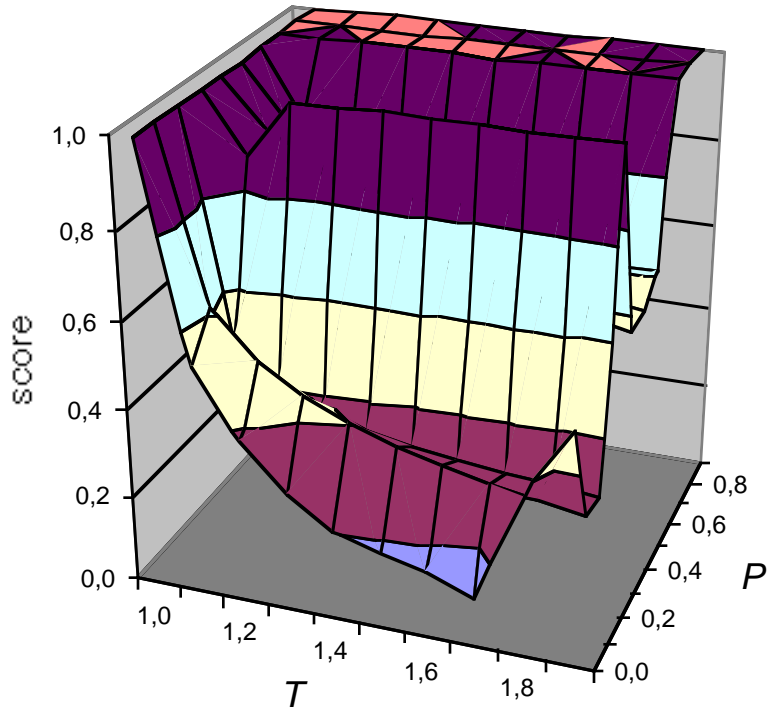


Figure 5. The average score for the 4-person PD as function of T and P is calculated from, in each point, the average score in the population in 10 simulations after a 60,000 generations transient.

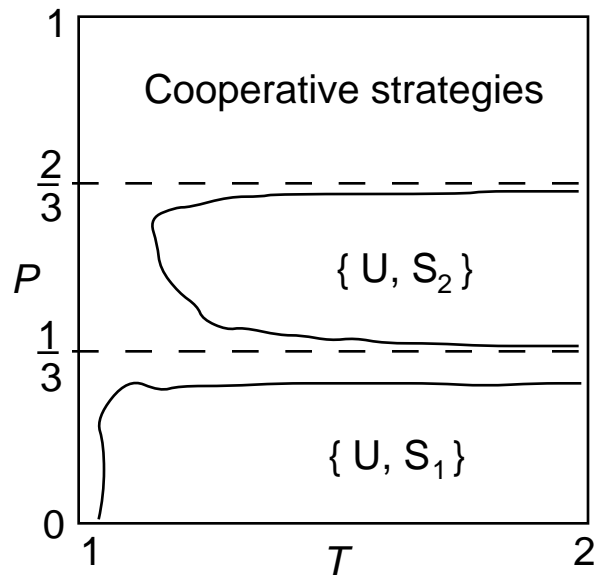


Figure 6. In this sketch of a phase diagram, based on Fig. 5 and simulation results, we illustrate how the parameter region (T, P) is divided into regions with cooperative behaviour and regions characterized by fixed points involving U and one of the S_k ($0 < k < n-1$). (We have not investigated the existence of other fixed points here, but, as was seen in Fig. 4, such exist.)

In Figs. 7-9, we illustrate how these regions change (for parameter kept constant, $T=1.5$) when the number of players n increases, from 3 to 5. When $n=2$ (not shown in the figures), we have the ordinary PD game, that has been extensively studied in various evolutionary models. In this case, cooperative behaviour is established by the coexistence of S_1 and S_0 , and the average score is close to 1 regardless of payoff parameters. (In this environment, S_1 behaves like Tit-for-tat.) It should be noted that the cooperative state is not evolutionarily stable, since genetic drift may alter the ratio between S_1 and S_0 such that a temporary increase of U may occur.

When the number of players n increases, the corresponding cooperative region (characterized by the absence of stable fixed points between two strategies) is approximately $(n-2)/(n-1) < P < 1$. This means that a high score for mutual defection actually increases the possibilities for cooperation. It is also worth noticing, that for some values of P (e.g., $P = 0.2$), increasing population size may lead to an increased level of cooperation, see Figs 7-9.

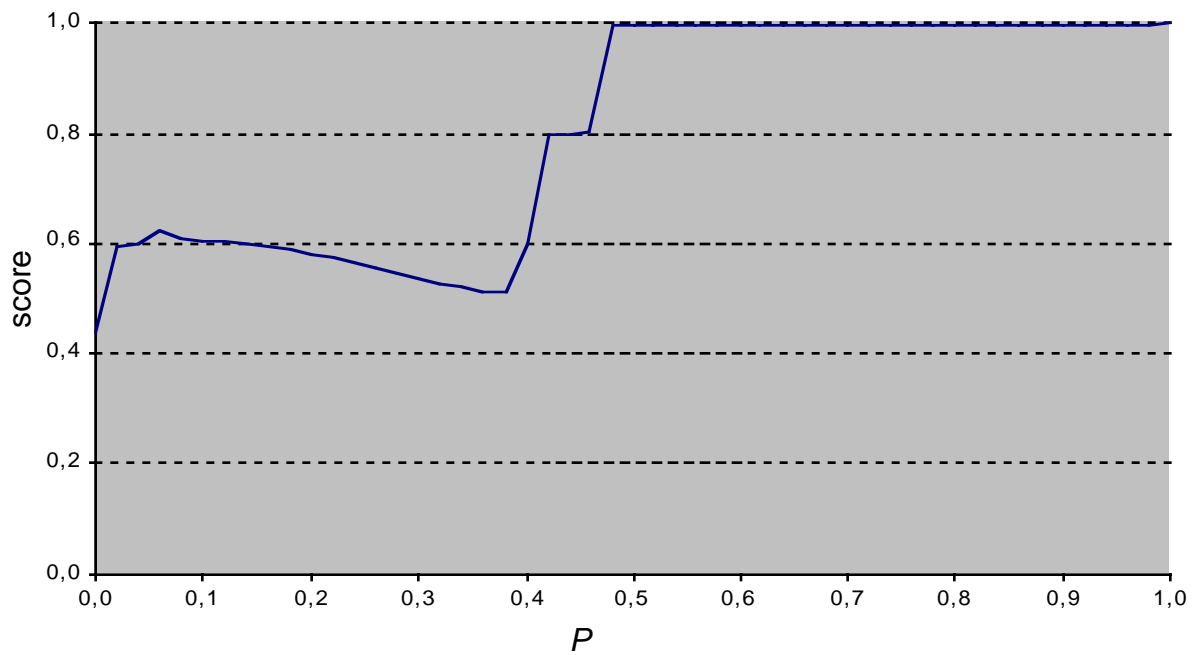


Figure 7. The average score as a function of P for the S_k strategies in the mean-field model using the 3-person PD game. The average is calculated from 10 simulations using 100,000 generations following a transient of 50,000 generations. Other parameter values are $T=1.5$, $p_{\text{mut}}=0.0001$, and $d=0.04$.

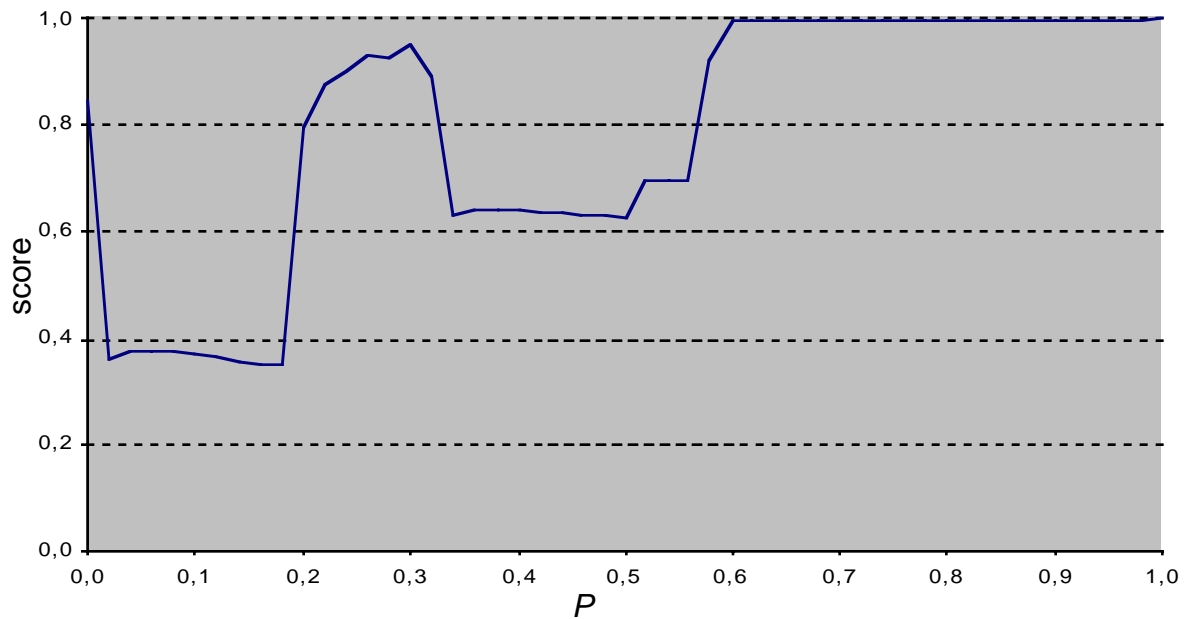


Figure 8. The average score as a function of P for the S_k strategies in the mean-field model using the 4-person PD game. The average is calculated from 5 simulations using 200,000 generations following a transient of 100,000 generations. Other parameter values are $T=1.5$, $p_{\text{mut}}=0.0001$, and $d=0.04$.

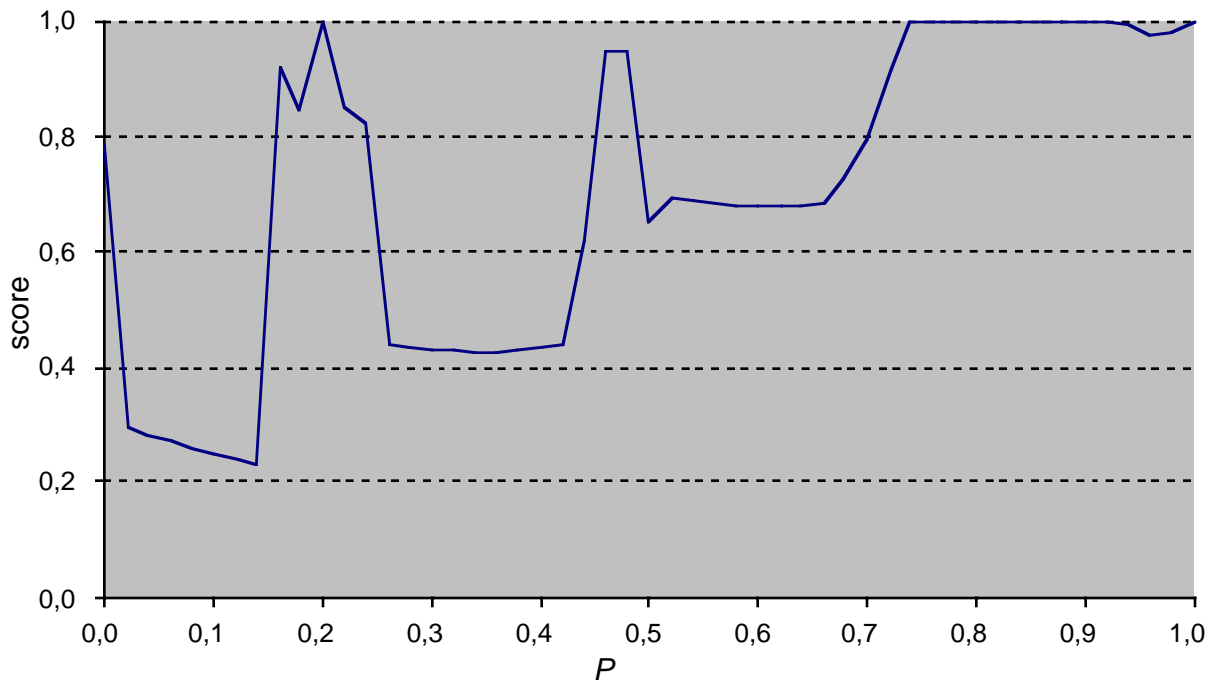


Figure 9. The average score as a function of P for the S_k strategies in the mean-field model using the 5-person PD game. The average is calculated from 5 simulations using 200,000 generations following a transient of 100,000 generations. Other parameter values are $T=1.5$, $p_{\text{mut}}=0.0001$, and $d=0.04$.

The change in cooperation level as P increases can be understood in the following way. First, for the regions where fixed points dominate the dynamics (less cooperative regions), we can use the analysis by Molander²³. There exists a stable equilibrium distribution consisting of unconditional defectors (U) and individuals using strategy S_{k^*} , where k^* is the lowest number $k < n - 1$ such that $V(C|k) > V(D|0)$. With n players in each group this condition can be written $k > (n-1)P$. For groups of four players $k^* = 1$ for $P < 1/3$ and $k^* = 2$ for $1/3 < P < 2/3$. (For $P > 2/3$, the lowest k equals $n - 1$, and there is no stable coexistence.) These different S_{k^*} strategies correspond to different average scores of the less cooperative regions. Secondly, for the cooperative regions in between, the fixed point given by the analysis above is weakend, and the mutations may sustain a distribution of strategies that destabilises the fixed point. This may result in either a distribution of cooperating strategies (which is not a fixed point), or a fixed point involving more than two strategies (as was observed in Fig. 4).

CA model with the simple strategies

If the S_k -strategies are put on a lattice according to the coevolutionary CA dynamics described above, the locality of the interactions completely changes the behaviour. It has been observed for the 2-PD game that cooperative behaviour is favoured in the CA dynamics compared to the mean-field case^{6,8}. Here we find that in the interior of the parameter region ($1 < T < 2$, $0 < P < 1$) the cooperative strategies dominate and the U strategy only occurs in minor bursts following a

mutation. Depending on the payoff parameters, U may expand locally by exploiting S_k strategies (with $k < n - 1$), but when areas of S_{n-1} strategies are met, the U strategy will disappear, for any values of (P, T) . This leads to the aggregated population dynamics pattern showing a quick shift from a population of defectors to a population dominated almost completely by cooperating strategies. In almost all of the parameter region, it is S_{n-1} that dominates. The spatial extension of the system does not allow for the type of coexistence between cooperative and defective strategies that the mean-field case exhibited in the form of stable fixed points. In principle, the spatial dynamics *may* give rise to spatio-temporal patterns that globally stabilizes a mixture of cooperators and defectors, as we shall see below, but it does not seem to be the case for the very limited strategy set used here.

Finite automata strategies

In order to illustrate the limitations of the previous strategy set and to get a flavour of the more complex evolutionary dynamics that a more open strategy set may generate, we show a few examples from the mean-field and the CA models using finite automata strategies. These are only illustrations that are included for a qualitative discussion of the FA strategies in a coevolutionary context. A thorough investigation of these strategies is being done and will be presented elsewhere.

The simulation of Fig. 10, based on the 3-player PD, uses the mean-field dynamics and illustrate how coevolution in a number of steps increases the level of cooperation in the system. The initial state consists of defectors only, but since there are 73 two-node FA that correspond to the unconditional defector, all of them are put in the group labeled U. At the end of the simulation, most of the strategies use three nodes, and the dominating strategy is cooperative in groups where both the opponents are.

For the simple strategy set the cellular automaton dynamics made cooperation extremely simple. This is also the case for finite automata, which is illustrated in Fig. 11. The transient from the initial state with unconditional defectors may involve several non-trivial strategies involved in various types of spatio-temporal patterns. At the end the population is mainly cooperating, dominated by a simple strategy that continues with cooperation as long as one or all of its neighbours defect.

For higher values of P , it seems, though, that cooperation is much harder to achieve than in the more limited class of the S_k strategies. Both the spatio-temporal behaviour and the dependence of payoff parameters will be further discussed in our future work on these models.

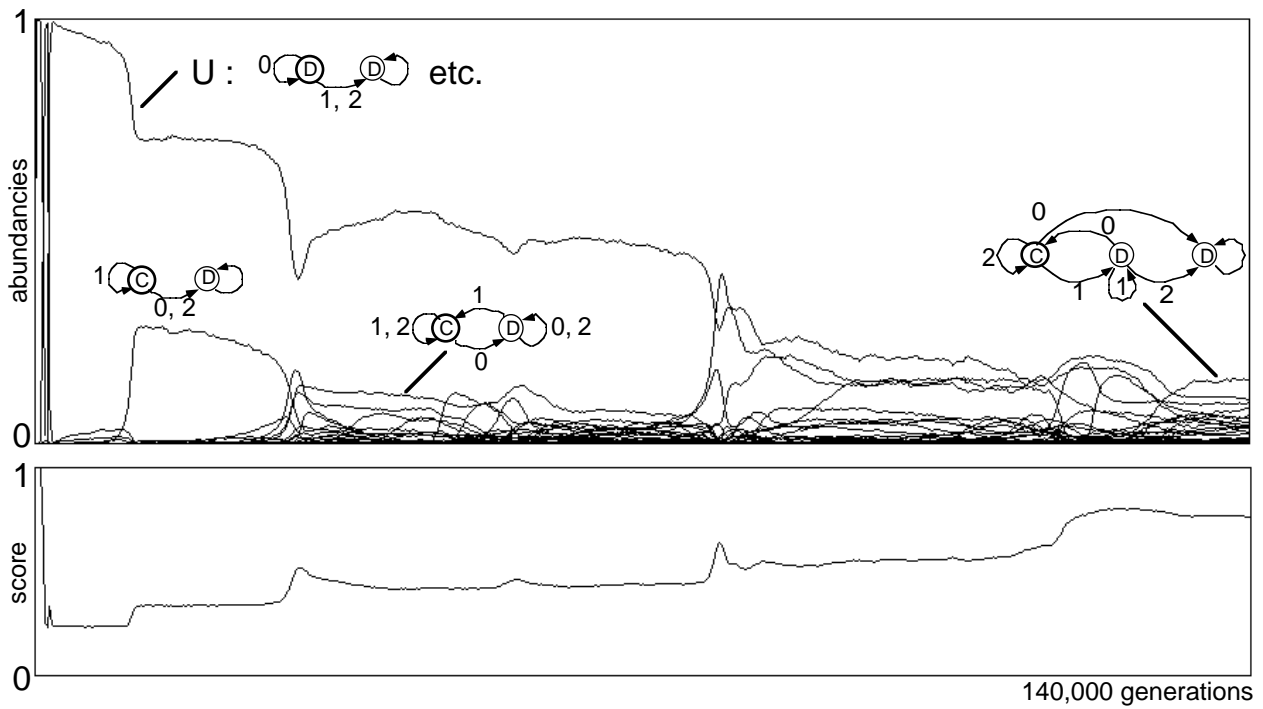


Figure 10. The coevolution of FA strategies in the mean-field $n=3$ PD game. The top chart shows how the initial population of defectors is gradually replaced by more cooperative strategies, but in this simulation there is still some free-riders left that manages to exploit the cooperative strategies. The bottom chart shows the increase in average score. Parameter values are $T=1.5$, $P=0.25$, $p_{\text{mut}}=5 \cdot 10^{-5}$, $d=0.05$, and the number of nodes is maximized to 3.

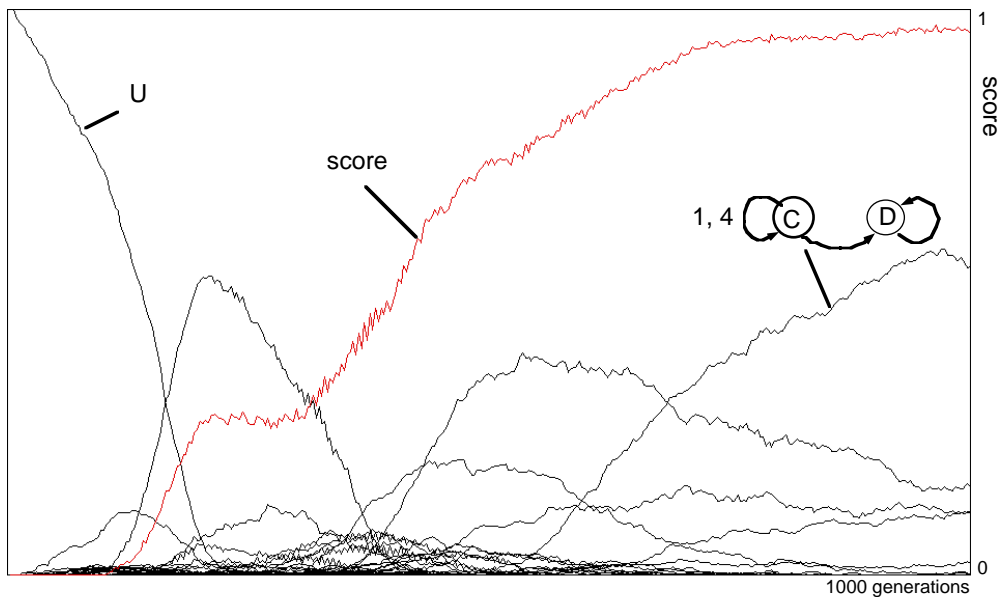


Figure 11. In this illustration of the cellular automaton model the system evolves from an initial state of only defectors to an almost fully cooperating population within a few thousand generations. Here $T = 1.5$, $P = 0.25$, and $p_{\text{mut}}=0.002$. The final strategy cooperates as long as one or all its neighbours did last round (otherwise it switches to unconditional defection). There are several possible strategies that may bring the population to the cooperating state.

Summary and possible extensions

In this paper we have discussed some aspects of coevolutionary models of the multi-person Prisoner's Dilemma game. In particular we have focused on a very simple strategy set that has previously been analysed mathematically in the literature. It is found that the coevolutionary dynamics in some cases may avoid being trapped by stable fixed points involving also defecting strategies, and in that way may establish a more cooperative behaviour. The structure of the "phase space" also implies that there are payoff parameters for which an increase in number of players in the PD group increases the chances for cooperative behaviour to evolve. Another non-intuitive result is that cooperation can be easier to achieve when the score P for mutual defection is large enough.

In the CA model, the introduction of a spatial dimension and the use of local interactions leads to the evolution of more cooperative strategies, as should be expected. For the simple strategy set S_k , we find that in almost all of the studied payoff region the strategy S_{n-1} dominates, i.e., the strategy that requires that all others in group cooperate for itself to cooperate. This effectively keeps out mutant unconditional defectors.

The finite automata strategy representation is discussed in the coevolutionary model context in a qualitative way. The increased capacity gained by this representation may be used both by advanced free riders as well as by more cautious cooperators, and it is not clear to what extent cooperative behaviour evolves in the different model types.

An immediate extension that would be interesting to try is the introduction of mistakes or misunderstanding. In the two-person PD game this has been used as one complication in order to increase the selection pressure and hence evolutionary activity.

The games studied here have all been "few-person" PD games. In principle the models can be extended to arbitrarily large games, but for practical (computational) reasons one is limited to a number of players in the order of 10. To be able to study more general problems of "the tragedy of the commons" type in coevolutionary contexts, one need to find both a suitable extension of the game and an appropriate *simple* strategy set. Some studies along these lines are in progress, involving harvesting and prey population dynamics.

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